Recursion - 6

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Solution of Non Homo Recc. Relation with const. coeffs.

Let $f(n) + c_1 f(n-1) + c_2 f(n-2) + \dots + c_k f(n-k) = q(n)$ —— (1)

where $q(n) \neq 0$ and c_i's are constant.

case II : $q(n) = q_0 + q_1 n$, (linear polynomial)

For particular solution we take $f(n) = d_0 + d_1 n$ in (1)

$$(d_0 + d_1 n) + c_1 (d_0 + d_1(n-1)) + c_2 (d_0 + d_1(n-2)) + \dots + c_k (d_0 + d_1(n-k)) = q_0 + q_1 n$$

$$(1 + c_1 + c_2 + \dots + c_k) d_0 + (n + c_1(n-1) + c_2(n-2) + \dots + c_k(n-k)) d_1 = q_0 + q_1 n$$

equate const term and coeff of n to get d_0 and d_1.

If the assumption for the particular solution contains the terms similar to terms present in homo. soln then modify assumption for particular soln by multiply with n.

Q:→ Solve the reccurence relation

$$f(n) - 7f(n-1) + 10f(n-2) = 6 + 8n$$

sol:→ Given recc relation is non homo. linear recc. relation with const. coeffs.

$$f(n) - 7f(n-1) + 10f(n-2) = 6 + 8n \quad \text{—— (1)}$$

Homo Saln $(f^h(n))$

Associated Homo. eqn $f(n) - 7f(n-1) + 10f(n-2) = 0$ —— (2)

For char eqn, take $f(n) = a^n$ in (2)

$$a^n - 7a^{n-1} + 10a^{n-2} = 0$$

$$\Rightarrow a^2 - 7a + 10 = 0$$

$$\Rightarrow \quad (a-5)(a-2)=0$$
$$\Rightarrow \quad a = 2, 5$$
$$\therefore \quad f^h(n) = A(2)^n + B(5)^n$$

Particular Soln ($f^p(n)$)

As $g(n) = 6+8n$

For particular soln,

Take $f(n) = d_0 + d_1 n$ in ①

$$f(n) - 7f(n-1) + 10f(n-2) = 6+8n$$
$$(d_0 + d_1 n) - 7(d_0 + d_1(n-1)) + 10(d_0 + d_1(n-2)) = 6+8n$$
$$(1-7+10)d_0 + (n - 7(n-1) + 10(n-2))d_1 = 6+8n$$
$$4d_0 + (n - 7n + 7 + 10n - 20)d_1 = 6+8n$$
$$4d_0 + (4n-13)d_1 = 6+8n$$
$$\underbrace{4d_0 - 13d_1} + \underbrace{4d_1 n} = 6+8n$$

Equate constant term and the coeff of n

$$4d_0 - 13d_1 = 6 \quad \Rightarrow \quad d_0 = 8$$
$$4d_1 = 8 \quad \Rightarrow \quad d_1 = 2$$

$$\therefore \quad f^p(n) = d_0 + d_1 n = 8 + 2n$$

Complete solution

$$f(n) = f^h(n) + f^p(n)$$
$$= A(2)^n + B(5)^n + 8 + 2n \quad \underline{\text{Ans}}$$

Q:→ Solve the reccurence relation

$$a_{n+3} - 3a_{n+2} + 3a_{n+1} - a_n = 24(n+2)$$

sol:→ Given recc. relation is non homo. linear recc. relation with constant coeffs.

$$a_{n+3} - 3a_{n+2} + 3a_{n+1} - a_n = 24n+48 \quad \text{(1)}$$

Homo saln (a_n^h)

Associated Homo eqn $\quad a_{n+3} - 3a_{n+2} + 3a_{n+1} - a_n = 0 \quad \text{(2)}$

For char eqn, take $a_n = a^n$ in (2)

$$a^{n+3} - 3a^{n+2} + 3a^{n+1} - a^n = 0$$

$$\Rightarrow \quad a^3 - 3a^2 + 3a - 1 = 0$$

$$\Rightarrow \quad (a-1)^3 = 0$$

$$\Rightarrow \quad a = 1, 1, 1$$

$$a_n^h = (A_1 + nA_2 + n^2A_3)(1)^n = A_1 + nA_2 + n^2A_3$$

Particular Solution (a_n^p)

As $q(n) = 24n+48$ which is a linear polynomial.

For particular soln,

i) Take $a_n = d_0 + d_1 n$, Then its similar terms are already present in homo soln. We have to modify it

ii) Take $a_n = d_0 n + d_1 n^2$, then its similar terms are already present in homo. soln. We have to modify it.

iii) Take $a_n = d_0 n^2 + d_1 n^3$, but similar term of $d_0 n^2$ is already present in homo soln. We have to modify it.

iv) Take $a_n = d_0 n^3 + d_1 n^4$ in (1)

$$a_{n+3} - 3a_{n+2} + 3a_{n+1} - a_n = 24n+48$$

$$\Rightarrow [d_0(n+3)^3 + d_1(n+3)^4] - 3[d_0(n+2)^3 + d_1(n+2)^4]$$
$$+ 3[d_0(n+1)^3 + d_1(n+1)^4] - [d_0 n^3 + d_1 n^4] = 24n+48$$

$\Rightarrow [(n+3)^3 - 3(n+2)^3 + 3(n+1)^3 - n^3] d_0$
$+ [(n+3)^4 - 3(n+2)^4 + 3(n+1)^4 - n^4] d_1 = 24n + 48$

By Binomial expansion

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^3$

$\Rightarrow 24d_1 n + 36d_1 + d_0 = 24n + 48$

Equating const. term and coeff of n,

$24d_1 = 24 \Rightarrow d_1 = 1$

$36d_1 + d_0 = 48 \Rightarrow d_0 = 12$

$\therefore a_n^p = d_0 n^3 + d_1 n^4 = 12n^3 + n^4$

Complete soln

$a_n = a_n^h + a_n^p = A_1 + nA_2 + n^2A_3 + 12n^3 + n^4$ Ans

Remark :→

If 1 occurs as char root then we're to modify our assumption for particular soln of $q(n) = q_0 + q_1 n$ (linear polyn) by multiplying with n^m where m is the multiplicity of 1 in char roots.